Reduction in Quantum Mechanics

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1. The reduction of classical mechanics to quantum mechanics

A fundamental problem in philosophy of science is how one theory is related to another.[1] In its diachronic aspect this raises the question of correspondence relations between an old theory S and a new theory L which replaces it. In what sense can the successful parts of S be captured, perhaps in some limiting sense, by the L-theory? If this is possible then the S-theory can be regarded in some suitable approximation, perhaps, as surviving alongside the L-theory. In the case where we have two contemporaneous theories in a correspondence relation, it is usual to talk in terms of reduction of the S-theory to the L-theory. In the case of quantum mechanics (QM) it arose as a replacement for classical mechanics (CM). So is there a sense in which CM reduces to QM? This is the problem of the classical limit of QM. It is not just a question of putting the reduced Planck's constant $\hbar$ equal to zero. The solutions of QM problems typically exhibit complicated singular behaviour as $\hbar$ tends to zero. Some features go smoothly over into classical behaviour, while others do not.

[~~As a simple illustration of a related problem consider the limit of~~ the wave equation

$$\frac{\partial^2\phi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 0 \qquad (1)$$

as c tends to ∞.

The equation itself becomes

$$\frac{\partial^2\phi}{\partial x^2} = 0 \qquad (2)$$

Skew (1)

But how are the solutions of (1) and (2) related? Suppose ϕ is just the displacement of a stretched string of length L, with fixed end points. The solution of (2) is then $\phi = 0$, but now look at the solution of (1), subject to the same boundary conditions, as c tends to ∞. For any finite c we have pulses travelling to right and left, reflected with 180° change of phase at each fixed end. Take the case of a pulse of unit amplitude of length $\ell < L$. It takes a time ℓ/c to pass any fixed point on the string. And it is easy to see that the time average of the displacement over a resolution time T ($>\ell/c$) can never exceed ℓ/Tc (remembering that the reflected pulses are always out of phase). For any given T, this time average goes to zero as c tends to ∞, and this is the sense in which the solution $\phi = 0$ is recovered in a limiting sense. But note also that for any c, however large, we can always chose a T ~~sufficiently small, to reveal the criss-crossing pulses on the string.~~

Of course, if (1) is regarded as the true L-theory, c is a fixed parameter. What our analysis shows then is that S-theory behaviour can be extracted by making T sufficiently large, i.e. it is a coarse-grained feature of the L-theory. A more fine-grained inspection will reveal departures from S-behaviour. It is in this pragmatic sense that CM can be regarded as reducible to QM. But philosophers are generally not interested in reductions of this pragmatic sort. They want to know, from an ontological or God's eye point of view, whether classical behaviour of quantal systems is ever exactly right. This is essentially the measurement problem of QM. For all practical purposes, as Bell was fond of saying, pointers behave as though they showed definite readings, but strictly and ontologically speaking, straightforward application of the QM formalism, interpreted according to orthodoxy, shows that pointers do not always show

definite readings, cats are not always alive or dead, and so on.

I do not, in this paper, intend to discuss in any further detail this particular problem of reduction, of how to extract classical behaviour out of quantum mechanics, but I will turn to a different, although undoubtedly related problem, of whether it is possible in QM to reduce the behaviour of complex systems in terms of the behaviour of their constituent parts.

But first I will make some general remarks about what I will call metaphysical reduction.

2. Metaphysical Reduction

Metaphysical reduction seeks to show that the ontology of the reduced S-theory and the physical laws governing the behaviour of the items comprising the S-ontology, can all be derived, under a suitable scheme of translation, from the ontology and the physical laws appropriate to the reducing L-theory. Let us consider a simple example of a supposed reduction, to illustrate the problems which arise in fleshing out this simple scheme. Suppose we want to claim that electrostatics reduces to electromagnetism. If we set the magnetic induction $\underset{\sim}{B}$ equal to zero in Maxwell's free-field equations, we certainly obtain the electro~~magnetic~~static equations for the electric field $\underset{\sim}{E}$, so are electrostatic fields just a special case of electromagnetic fields? The answer is decisively, no. The electromagnetic field can be thought of as an ordered pair $(\underset{\sim}{E},\underset{\sim}{B})$. The special fields of the form $(\underset{\sim}{E},0)$ are quite distinct from the electrostatic fields $\underset{\sim}{E}$, which have the same form as the first member of the ordered pair $(\underset{\sim}{E},0)$. More formally the electrostatic field structure is embeddable into the electromagnetic structure - it is not, however, a substructure. In other words electrostatic fields are mapped into electromagnetic fields, in

shove (2)

effecting the proposed reduction, they are not just a special case of them. In this example the embedding map in question is the formal counterpart of what I called in my preliminary formulation of metaphysical reduction, 'a suitable scheme of translation'.

But, from the point of view of the L-theory, there just are no electrostatic fields in the sense that the S-theorist intended, i.e. divorced from all questions of how the magnetic induction is behaving. The L-theorist would say that when the S-theorist used the term electrostatic field, the sense of the term was quite distinct from that of any special sort of electromagnetic field, but the reference, unbeknownst to him (the S-theorist) was the special sort of electromagnetic field that L-theorists are talking about when they refer to electrostatic fields.

We are following here a version of the causal theory of reference made popular by Kripke and early embodiments of Putnam. Prescientific talk of water actually refers to the H_2O-structured substance that we know, or rather claim to know, was causally responsible for the prescientific talk. To take another familiar example, the temperature of a gas in thermodynamics is really referring to the mean kinetic energy of the molecules. Temperature in thermodynamics is a quite different concept from "mean kinetic energy of molecules", but these two concepts are coextensional, a coextensionality which is law-like, but contingent, in the sense that it is logically possible for hot bodies to possess no motion in their parts at all, although this is not physically possible. Already I have begun to skate uncritically over some pretty thin philosophical ice! Sense, reference, translation, possibility, necessity, these are the very stuff of philosophical controversy, but I think I have said enough to show where some of the puzzles about metaphysical reduction originate, viz. in

the innocent-sounding phrase "a suitable scheme of translation".

Before turning to the specific problems of whole-part reductions and the particular case of quantum mechanics, I want to say something briefly about another aspect of the preliminary formulation. I said that the behaviour of the S-items could be derived from the behaviour of the L-items. This suggests some sort of logical derivation between linguistically presented L- and S-theories under, of course, some scheme of translation. Modern discussions of intertheory relations have usually followed model-theoretic formulations.[2] What we require for reduction is that it is not possible for two distinct S-models to correspond (under our scheme of translation) to a unique L-model. In other words L-behaviour fixes S-behaviour. A still more general way of expressing this idea, that is philosophically fashionable, is in terms of supervenience.[3] For reduction S-behaviour must supervene on L-behaviour.

3. Part-whole reductions

Many of the proposed reductions in the physical sciences involve a micro-reduction, i.e. the reduction of macrophysical objects and their complex behaviour to their microphysical constituents and the supposedly simpler principles governing the behaviour of these constituents. This is after all the basic idea behind 'atomic theories' taken in a general sense. Such reductions are claimed to be not only metaphysically 'true', but also explanatory in the sense of providing an ultimate causal account of macroscopic phenomena, and/or achieving a theoretical unification in the sense that many diverse macroscopic phenomena can be explained in terms of a few simple laws governing the micro-constituents.[4] Thus, in the paradigm example of the kinetic theory of gases, many apparently unrelated

phenomena, such as viscosity, diffusion, thermal conductivity, specific heats, etc., are all explained in terms of molecular motions and collisions. Gases are in truth nothing but collections of molecules in random motion. Or, to take the famous example of Eddington,[5] the table in front of me is not really the solid block of wood familiar to everyday experience, but is in fact the table of modern theoretical physics, mostly empty space, interspersed with atoms, which in their turn are resolved in terms of nucleons and electrons, and the nucleons are resolved in terms of quarks, and so on.

While this view was roundly attacked by the philosopher Susan Stebbing,[6] our previous analysis suggests that while the everyday sense of 'table' is certainly not captured by the theoretical physics' reduction, nevertheless the reference of 'table' really is the collection of elementary particles, executing suitable microphysical behaviour, just as Eddington claimed.

But what we have said about the micro-reduction of tables shows that, from the explanatory point of view, there are many levels at which the reduction can be taken, we don't always have to go to the 'ultimate' constituents in every case of microphysical explanation. The reduction of biology to chemistry can be effected at the molecular level, the reduction of chemistry to physics at the atomic level, while the more exotic reductions in terms of so-called elementary particles, may only be relevant to physics itself. But from the metaphysical as distinct from the explanatory point of view, it is with the ultimate constituents that the 'nothing but' claims need to be examined. This raises familiar problems What are the ultimate constituents? Are we faced with a never-ending succession of 'chinese boxes', each level revealed at an appropriate energy

of experimental investigation? Are we dealing with constituent particles at all? Or is the language of quantum field theory appropriate to answering the 'nothing but' question, or superstrings or whatever?

The best we can do in dealing with such problems is to take currently accepted physical theory as the basis for answering the 'nothing but' questions. In what follows we shall confine ourselves to the non-relativistic quantum mechanics of point particles, such as electrons, in trying to assess the credentials of the part-whole reduction programme in modern theoretical physics. The problems we shall expose will, no doubt, survive in any of the more realistically accurate relativistic and field-theoretic extensions of QM.

But first of all we shall make some general remarks on how part-whole reductions can be expected to work in situations such as classical atomic theories, where the particular problems we shall meet in the case of QM do not arise.[7] We shall be concerned with parts, the atomic constituents, which are spatially separated. We shall not be concerned then with the more general analysis of wholes into parts, the subject of mereology, where spatial separation of the parts need not obtain. (Think of resolution of a force into its components, the analysis of the motion of a stretched string in terms of harmonics, or more abstractly the sense in which one could regard the number five as composed ~~to~~ of the numbers two and three.) Reverting then to the classical atomic physics example~~s~~, in what sense is it true that the properties of a collection of atoms supervene~~s~~ on the properties of the constituent atoms? It is easy to trivialize the supervenience relation if we are too generous in what we count as a property of the constituent atoms. Suppose P is some arbitrary property of the collection of atoms, then if we allow as a property R of the

constituent atoms that they combine to form a collection with the property P, then P trivially supervenes on the property R of the constituents. To make an interesting thesis of reduction, we might try something along the following lines:

> All the intinsic properties (monadic and relational) of a collection of atoms supervene on the intrinsic monadic properties of the constituent atoms together with the spatial relations between the atoms.

Show (3)

This is the sort of reduction thesis which would seem to be true of classical atomic theories. But notice that, although specifying the intrinsic monadic properties, such as the masses, of the atoms together with their spatial relations does serve to fix all the intrinsic properties of the collection of atoms, it would be definitely misleading to think of the collection as 'nothing but' the individual atoms and their spatial relations. It would be preferable, from the metaphysical point of view, to regard the reduction as effected in terms of the constituent atoms together with their <u>causal</u> relations, as governed by the forces between the atoms. It is true that fixing the spatial relations will fix the forces, but this serves to emphasize that defining reduction in terms of supervenience may not bring out correctly the 'nothing but' aspect of reduction that we need intuitively to capture.

Let us specialize now to the simplest composite atomic system - a two-body system, where we will treat the atoms as point masses. What are the intrinsic monadic properties of the individual atoms? Well, first of all, there are the masses, which are independent of the dynamical state of the constituent atoms, and then there are the state-dependent properties represented by real-valued functions on the 6-dimensional phase spaces

Ω_1 and Ω_2 of the two atoms, labelled 1 and 2. All these state-dependent properties for each individual atom are thus determined by the characteristic function which picks out the precise location representing the state of that atom in its own phase space. In order to represent the state of the two-body system we employ a composite phase space $\Omega = \Omega_1 \times \Omega_2$, which is just the Cartesian product of Ω_1 and Ω_2. Knowing the precise state of atom 1 as located in Ω_1 and the precise state of atom 2 as located in Ω_2, this fixes the precise state of the composite system in $\Omega_1 \times \Omega_2$, and hence fixes all the properties of the composite system which are represented by real-valued function on Ω. (3) cont'd

Notice that since the spatial coordinates of atom 1 are included as part of the state specifications in Ω_1, and similarly for Ω_2, we have actually assimilated the spatial relation between the atoms to what we have chosen to call the intrinsic monadic properties of the atoms, so the spatial relation does not figure as an independent item in the supervenience basis, on this particular account.

Let us now see how this analysis needs to be modified if the two-body system is described by QM rather than CM.

4. Quantum Entanglement and Quantum Holism

In QM the maximally specific states of a single system are described by vectors in an appropriate Hilbert space, instead of locations in a phase space. For a composite two-body system the relevant state space is now $\mathcal{H}_1 \otimes \mathcal{H}_2$, where $\mathcal{H}_1$ and $\mathcal{H}_2$ are the individual Hilbert spaces, and $\otimes$ denotes the tensor product of the two spaces, replacing the Cartesian product $\Omega_1 \times \Omega_2$ in the classical case. If $\psi_1 \in \mathcal{H}_1$ is a possible state vector for system 1, and $\psi_2 \in \mathcal{H}_2$ is a possible show (4)

state vector for system 2, then $\psi_1 \otimes \psi_2 \in \mathcal{H}_1 \otimes \mathcal{H}_2$ is a possible state for the composite system. In the orthodox interpretation of QM the system 1 is regarded as possessing a value for the observable Q_1, iff the state of the system is an eigenstate of Q_1. Suppose ψ_1 is such an eigenvector with eigenvalue q_1, then in state ψ_1 we can say, according to orthodoxy, that system 1 has the property that Q_1 has the value of q_1. Similarly if ψ_2 is an eigenvector of Q_2 with eigenvalue q_2, then in the state ψ_2 system 2 has the property that Q_2 has the value q_2. Now consider the state $\psi_1 \otimes \psi_2$ for the composite system. This is both an eigenstate of Q_1 with eigenvalue q_1 and of Q_2 with eigenvalue q_2, so in this state we can say that the composite system has the property that Q_1 has the value q_1 and Q_2 has the value q_2. Such a property clearly supervenes on the properties for the individual systems of having the value q_1 for Q_1 for system 1 and having the value q_2 for Q_2 for system 2. For this kind of composite property which arises in the case where the state of the composite system is itself a tensor product of individusal state vectors the part-whole reduction works in essentially the same way as it does in the classical case. But we now come to the decisive point. There are many vectors in $\mathcal{H}_1 \otimes \mathcal{H}_2$ which cannot themselves be represented as the tensor product of vectors in $\mathcal{H}_1$ and $\mathcal{H}_2$ respectively, but only as superpositions of such tensor products. Such states are generally known as entangled states. Thus suppose ψ_1' is another eigenvector of Q_1 with eigenvalue q_1', and ψ_2' is another eigenvector of Q_2 with eigenvalue q_2'. Then consider the state of the composite system $\Psi = 1/\sqrt{2}$ $(\psi_1 \otimes \psi'_2 + \psi_1' \otimes \psi_2)$. This is certainly not an eigenvector of Q_1 or Q_2 separately, but suppose that $q_1 + q_2' = q_1' + q_2 = \alpha$ say, then Ψ is an eigenvector of $Q_1 + Q_2$ with eigenvalue α. So the composite system

now has the property that $Q_1 + Q_2$ has the value λ, but this cannot supervene on any properties ascribing values to Q_1 and Q_2 separately, because on the orthodox interpretation there are no such properties. So the part-whole reduction has failed in the sense that there exist perfectly good properties of composite systems, which do not supervene on any properties of the constituent systems.

It might be thought that this failure of reduction could be circumvented in a hidden-variables approach to the interpretation of QM, where all observables are regarded as having definite values at all times. Surely the problem arose because we did not allow Q_1 and Q_2 to have definite values in entangled states such as $\tilde{\underline{\Psi}}$. In fact the situation in this type of interpretation with regard to part-whole reductionism is quite subtle. We shall sketch the relevant results, without going into technical details of the proofs which are readily available elsewhere.[8]

Realist construals of QM in which all observables possess sharp values at all times have met with two major problems. The first is posed by the Kochen-Specker paradox in which value assignments to appropriate observables subject to a constraint known as FUNC (Functional Composition Principle) which demands that value assignments respect functional relationships between observables, lead to an algebraic contradiction. The second problem arises from the work of Bell, which shows that in the case of two spatially separated systems, the correct QM correlations between the two systems can in general only be obtained by violating locality, i.e. by assuming that the value of an observable pertaining to one system depends on what sort of measurement procedure is performed on the other system.

In their (1983) Heywood and Redhead showed how these results are related by giving a demonstration of nonlocality which does not involve

consideration of correlation functions as in Bell's work, but consists in demonstrating a Kochen-Specker type of contradiction on one of two separated systems under the assumption of certain locality principles. ~~Specifically what was shown was the following:~~

Show (5) and (6)

$$\text{FUNC}^{*} + \text{VR} + \text{OLOC} + \text{ELOC} \implies \text{Contradiction}$$

where the four principles involved in deriving the contradiction will now be explained.

We use the following notation. For any observable Q we denote the value of Q in the QM state ψ by $[Q]^{\psi}$. Suppose B and C are two maximal (i.e. nondegenerate) observables and A is a third observable, which may not be maximal, such that the associated self-adjoint operators $\hat{B}$, $\hat{C}$ and $\hat{A}$ representing these observables in the mathematical Hilbert space formulation, satisfy the functional relationships $\hat{A} = f(\hat{B}) = g(\hat{C})$. Then we distinguish observables A_B, A_C where $[A_B]^{\psi} =_{df} f([B]^{\psi})$ and $[A_C] =_{df} g([C]^{\psi})$. The assumption that whe $\hat{B}$ and $\hat{C}$ do not commute (which requires A to be non-maximal) then the values $[A_b]^{\psi}$ and $[A_C]^{\psi}$ are in general distinct enables one to resolve the Kochen-Specker paradox without giving up realism.

We can now formulate

<u>FUNC*</u> : $[A_B]^{\psi} = [A_C]^{\psi}$ if $\hat{B}$ and $\hat{C}$ are maximal <u>commuting</u> operators.

This is a specialization of FUNC itself which says that the above equality holds even when $\hat{B}$ and $\hat{C}$ do not commute.

We now turn to the so-called Value Rule VR:

$$\underline{VR} : P_Q^{\psi}(\lambda) = 0 \Rightarrow [Q]^{\psi} \neq \lambda$$

Where $P_Q^{\psi}(\lambda)$ denotes the QM probability of finding the value λ on measuring the observable Q in the state ψ.

In order to understand OLOC and ELOC we first introduce two senses of <u>contextuality</u> for observables:

> <u>Ontological Contextuality</u> arises from distinguishing the magnitudes A_B and A_C in the way described above.
>
> <u>Environmental Contextuality</u> recognises that the values of these magnitudes may depend on the environment in the most general sense, in particular on the physical state of the apparatus set to measure some maximal observable on the system.

We apply these notions to two spatially separated systems and arrive at two quite distinct locality principles:

> <u>OLOC</u>: Locally maximal physical magnitudes n either of two spatially separated systems are not 'split' by ontological contextuality relative to the specification of different maximal physical magnitudes for the joint system.

> <u>ELOC</u>: The values possessed by a local physical magnitude cannot be changed by altering the arrangement of a <u>remote</u> piece of apparatus which forms <u>part</u> of the measurement context for the joint system.

Note that in terms of measurement results OLOC and ELOC cannot be distinguished. The violation of either demonstrates a dependence of the outcome recorded by an apparatus connected to one system on the setting of the apparatus connected to the other (remote) system.

But metaphysically speaking violating OLOC is quite different from violating ELOC. Violating OLOC means that we cannot specify all the locally maximal magnitudes independently of properties relating to the whole combined system. In such a situation part-whole reduction is obviated in the sense that one cannot even formulate the idea of such a reduction, unless the component systems possess their own local properties, independently of the holistic context. This is quite a different sense in which the part-whole valuation fails from that obtaining in the orthodox interpretation of QM, where there were not enough local properties to sustain the reduction. In the situation we are now contemplating there are in a sense too many 'local' properties, but due to the ontological contextuality these properties are not really local properties at all.

But our basic contradiction shows that we do not <u>need</u> to violate OLOC. Assuming that we refuse to give up the very plausible principles FUNC* and VR, we can still avoid the contradiction by retaining OLOC, and giving up ELOC. This amounts to admitting action-at-a-distance (even instantaneously) between genuinely local properties (since OLOC is maintained) of one system and the physical disposition of a remote piece of

apparatus. This is exactly the sort of nonlocality originally contemplated by Bell and provided a prima facie conflict with the constraints of special relativity, standardly understood as prohibiting superluminal causal interactions. In view of this situation, avoidance of the contradiction, modulo retaining FUNC* and VR, may be achieved in a more satisfactory way by giving up OLOC. But this means, as we have seen, giving up part-whole reductionism.

In passing we emphasise that giving up OLOC does not involve causal action-at-a-distance. The following simple analogy proposed by David Lewis may be helpful here. Lewis invites us to contemplate a bilocal hand, i.e. a hand which occupies two spatial configurations at the same time. A suitable bilocal hand would look just like a pair of ordinary hands. But suppose we are introduced to a person with a bilocal hand, and endeavour to shake his right hand. As we move his right hand up and down, his left hand would also move up and down in mysterious harmony, not because there is causal interaction between the two hands, but because the left hand just is a (bilocal) manifestation of the right hand, or perhaps more correctly of the unique hand, bilocally manifested, that this curiously handicapped person actually possesses.

We want finally to comment briefly on the claim that has been made in the literature that an analogy to violation of OLOC occurs in the case of stochastic hidden-variable theories which violate the so-called Jarrett completeness condition (also commonly referred to as outcome dependence).
~~To explain the terminology consider two quantities a and b for two~~ spatially separated systems α and β in a Bell-type experiment. Let $\mathcal{E}_a$ and $\mathcal{E}_b$ be possible values for these quantities. Let an apparatus A measure $\mathcal{E}_a$ on α and an apparatus B measure $\mathcal{E}_b$ on β. Let the two

Show (7) (8) and (9)

systems emerge from a source S characterized in its complete state by a parameter (hidden variable) $\underline{\lambda}$, with possible values ε_λ. Stochastic hidden-variable theories start by assuming the existence of a triple joint distribution

$$\text{Prob}_{\underline{a},\underline{b},\underline{\lambda}}^{\eta_A,\eta_B}(\varepsilon_a, \varepsilon_b, \varepsilon_\lambda)$$

for the probability that $\underline{a}$ has the value ε_a and $\underline{b}$ the value ε_b and $\underline{\lambda}$ the value ε_λ, where the indices η_A, η_B, indicate the configurations of the apparatus A and the apparatus B respectively.

Quite generally we can write:

$$\begin{aligned}\text{Prob}_{\underline{a},\underline{b},\underline{\lambda}}^{\eta_A,\eta_B}(\varepsilon_a,\varepsilon_b,\varepsilon_\lambda) &= \text{Prob}_{\underline{a}}^{\eta_A,\eta_B}(\varepsilon_a / \underline{b} = \varepsilon_b \ \& \ \underline{\lambda} = \varepsilon_\lambda) \\ &\times \text{Prob}_{\underline{b}}^{\eta_A,\eta_B}(\varepsilon_b / \underline{\lambda} = \varepsilon_\lambda) \\ &\times \text{Prob}_{\underline{\lambda}}^{\eta_A,\eta_B}(\varepsilon_\lambda)\end{aligned}$$

Now Jarrett showed in his (1984) that by making two assumptions the triple joint distribution could be expressed in a form which allowed the proof of the famous Bell inequality. Since this is violated by experiment, Jarrett concluded that one or other of the assumptions must be abandoned. The first assumption, Jarrett completeness or outcome independence, says:

$$\text{Prob}_{\underline{a}}^{\eta_A,\eta_B}\left(\varepsilon_a / \underline{b}=\varepsilon_b \,\&\, \underline{\lambda}=\varepsilon_\lambda\right) = \text{Prob}_{\underline{a}}^{\eta_A,\eta_B}\left(\varepsilon_a / \underline{\lambda}=\varepsilon_\lambda\right)$$

The second assumption, Jarrett locality or parameter independence, says that:

$$\text{Prob}_{\underline{a}}^{\eta_A,\eta_B}\left(\varepsilon_a / \underline{\lambda}=\varepsilon_\lambda\right) \quad \text{is independent of} \quad \eta_B ,$$

$$\text{that } \text{Prob}_{\underline{b}}^{\eta_A,\eta_B}\left(\varepsilon_b / \underline{\lambda}=\varepsilon_\lambda\right) \quad \text{is independent of} \quad \eta_A$$

$$\text{and } \text{Prob}_{\underline{\lambda}}^{\eta_A,\eta_B}\left(\varepsilon_\lambda\right) \quad \text{is independent of } \eta_A \text{ and } \eta_B .$$

Under these assumptions, the triple joint distribution reduces to the factorized form

$$\text{Prob}_{\underline{a}}^{\eta_A}\left(\varepsilon_a / \underline{\lambda}=\varepsilon_\lambda\right) \times \text{Prob}_{\underline{b}}^{\eta_B}\left(\varepsilon_b / \underline{\lambda}=\varepsilon_\lambda\right) \times \text{Prob}_{\underline{\lambda}}\left(\varepsilon_\lambda\right)$$

~~from which, as we have remarked, it is easy to derive the Bell inequality.~~

Now it is certainly true that violating parameter independence is perfectly analogous to violating the ELOC condition. But what about violating outcome dependence? Is this analogous to violating OLOC? In his 1989 paper on relational holism Paul Teller claims that violating outcome

dependence is not a case of direct action-at-a-distance between the events consisting of a taking the value $\mathcal{E}_a$ and b taking the value $\mathcal{E}_b$, essentially on the grounds that the individual outcomes are uncontrollable, so outcome dependence cannot, according to Teller, be exploited to achieve signalling.[10] This line of argument has been challenged by the present author,[11] who claims that outcome dependence does . meet all the conditions for stochastic causality imposed by a counterfactual analysis.

In point of fact OLOC is essentially already assumed in the formulation of outcome dependence in attributing the magnitudes a and b unambiguously to the individual component systems.

If we want to look for something more nearly holistic, but still quite different from a violation of OLOC, in a stochastic formulation, the present author has argued that the conditional probability $\mathrm{Prob}^{\Psi}(\underline{a} = \mathcal{E}_a \,/\, \underline{b} = \mathcal{E}_b)$ afforded by the QM formalism in the entangled state Ψ of the combined system is evidence for a harmony-at-a-distance (or passion-at-a-distance as it is sometimes called) as distinct from causal action-at-a-distance, at any rate under suitable interpretations of the state vector as comprehending a listing or catalogue of all the conditional probabilities appropriate to that state. If the supervenience basis in the part-whole reduction includes only causal relations between the individual systems, then such passion-at-a-distance cannot be regarded as reducible.

5. Conclusion

We have seen by our detailed analysis that QM provides good grounds, both on the orthodox and on the hidden-variable interpretations, for a holistic aspect that argues against the possibility of classical-type part-

whole reduction. The reductive hierarchy founding the whole of science on the properties of individual elementary particles is thus shown to be mistaken. This is most clearly seen in the orthodox Copenhagen insistence that in order to formulate a theory of microsystems, we must antecedently employ the classically described experimental arrangement of apparatus that allows the manifestation of the properties of these microsystems, while recognizing that the macroscopic apparatus is in one sense just made up of micro-constituents, but in another sense not reducible to them.

There is no doubt that the failure of what Teller refers to as local particularism is one of the most profound revisions in our ultimate metaphysical weltanschaung, that has been engendered by our most fundamental physical theory, viz. quantum mechanics.

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